期权策略的风险指标系统：Greeks的介绍与应用

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内容

- Option 101
- Greeks简介
- 期权策略简介
- 应用Greeks管理期权策略

Options 101

- Black 模型, 等价关系 Put-Call Parity
- 内在价值Intrinsic Value, 时间价值 Time Value
- 杠杆效应 Leverage Effect (Quiz)
- 合成定价 Synthetic pricing for European Option

Black 模 型

$$C(t) = e^{-r(T-t)}\left[F^{T_1}(t)N(d_1) - kN(d_2)\right]$$

$$d_1 = \frac{\ln\left(\frac{F^{T_1}(t)}{k}\right) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

等价关系 Put Call Parity

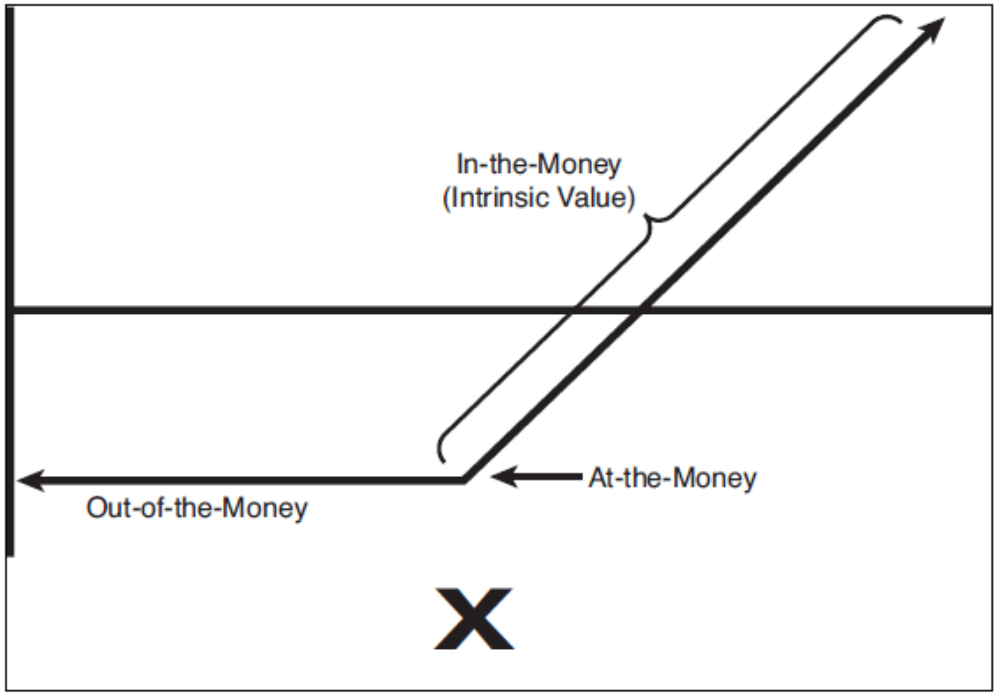
- Formula for put can be obtained from put-call parity

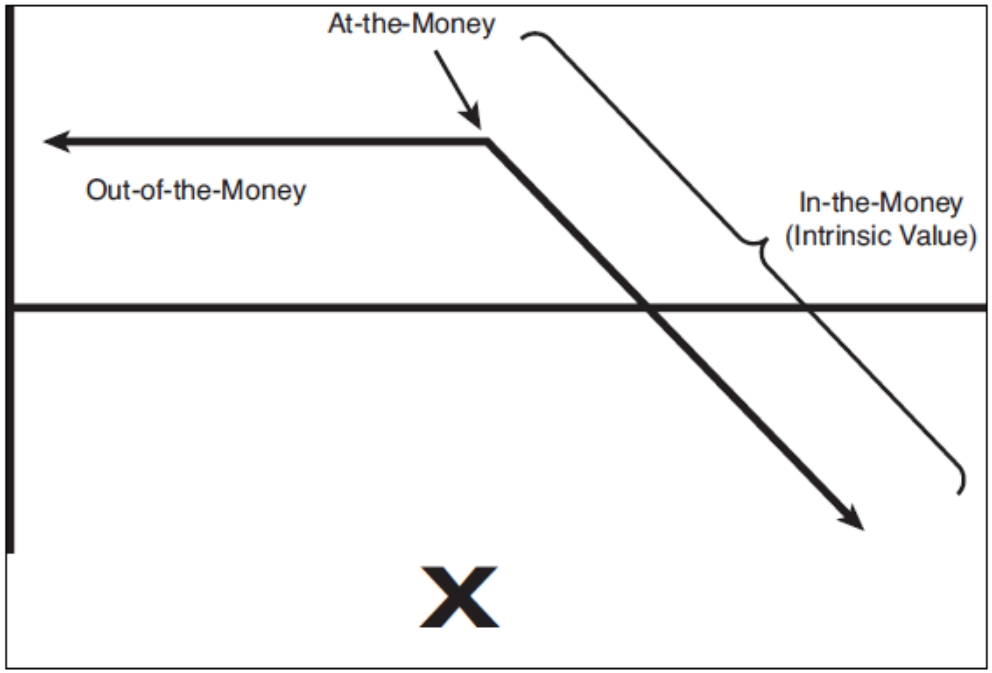
$$C(t)+ke^{-r(T-t)}=F^{T_1}(t)e^{-r(T-t)}+P(t)$$

- In fact, future price behaves like that of a stock paying a continuous dividend rate equal to r

内在价值 Intrinsic Value

- The amount that
  - Market price is higher than strike price for a call
  - Market price is lower than strike price for a put
- In-the-money
  - Future price is above/below strike price (call/put)
- At-the-money
  - Future price equals to the strike price
- Out-of-the-money
  - Future price is below/above strike price (call/put)


In-the-Money
(Intrinsic Value)
At-the-Money
Out-of-the-Money
X


At-the-Money
Out-of-the-Money
In-the-Money
(Intrinsic Value)
X

时间价值 Time Value

- We have intrinsic value of option, time value is the remaining part

- The option price can be decomposed as

$$C(t) = \text{Intrinsic value} + \text{Time value}$$

杠杆效应 Leverage effect

- From black model, it is easy to have $\frac{\delta C}{C} > \frac{\delta S}{S}$

- The return generated by money invested in the call is strictly higher than the return on underlying asset during the same period

提问 Quiz

- Which of the following long call options provides greater leverage?
  - A. 100 contracts costing $0.24/$10 out-of-the-money
  - B. 10 contracts costing $2.57/at-the-money

合成定价 Synthetic Pricing

- Asset or Nothing
- Cash or Nothing

Greeks简介

- Greeks, 期权风险度量系统
- Greeks 与影响期权价格的因素
  - 标的资产价格(delta, gamma)
  - 到期日(theta)
  - 波动率 (vega)
  - 无风险利率(rho)

Delta

- 期权价格对标的资产价格的偏导数
- 度量了期权价值对标的资产价格变化的敏感程度
- 其定义为

$$\Delta = \frac{\partial \Pi}{\partial S}$$

- 根据Black-Scholes公式，欧式期权的delta是:

$$\Delta_c = e^{-qT} N(d_1), \quad \Delta_p = e^{-qT} N(d_1) - 1$$

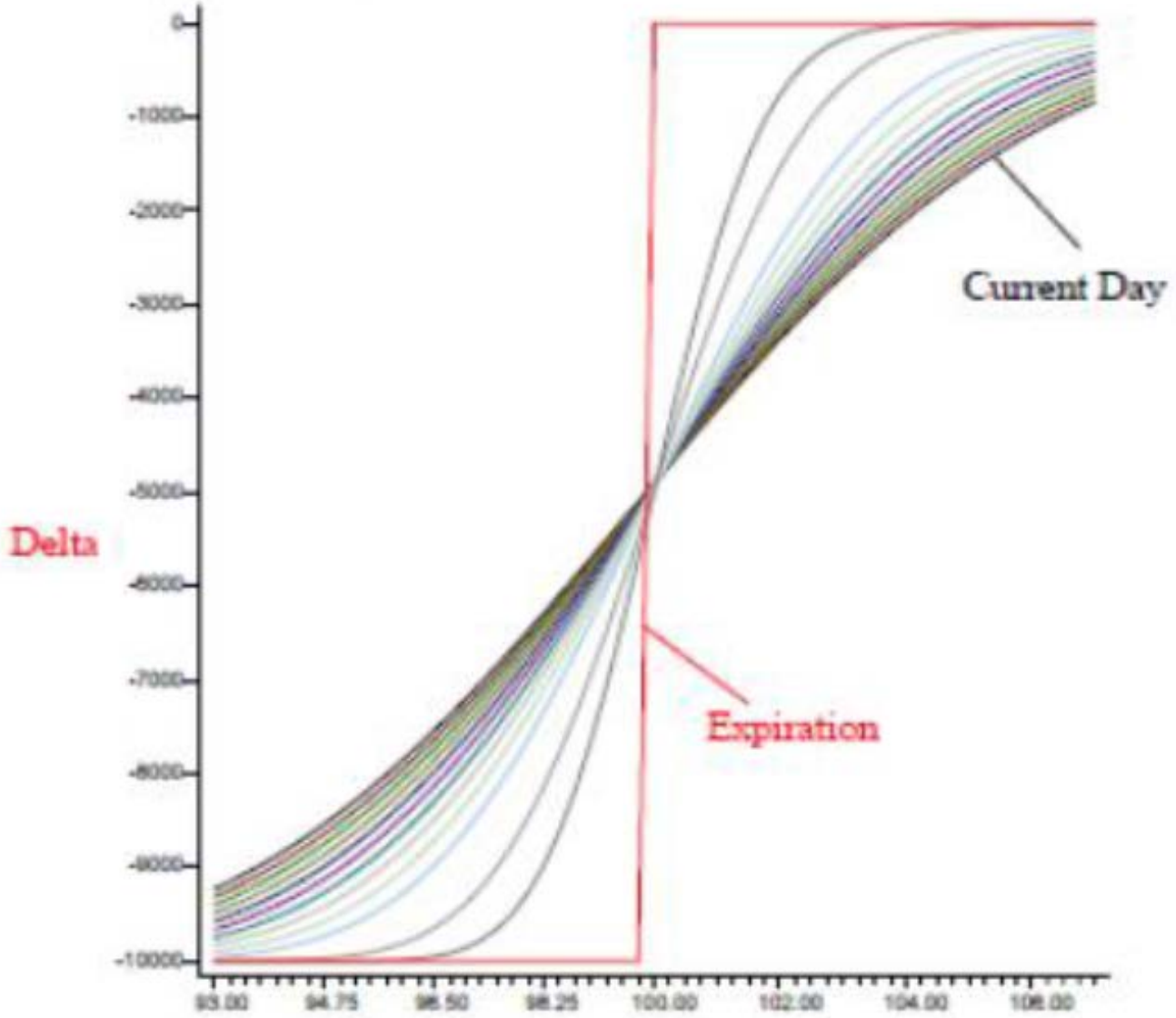
Δ受标的资产价格和时间变化的影响

- 标的资产价格对Δ的影响
  - 平值的看跌期权的 Δ 接近 -0.5
  - 当看跌期权变为深度实值期权时，Δ 趋于-1
  - 当看跌期权变为深度虚值期权时，Δ 趋于0

  - 平值的看涨期权的 Δ 接近 0.5
  - 当看涨期权变为深度实值期权时，Δ 趋于1
  - 当看涨期权变为深度虚值期权时，Δ 趋于0

- 越临近到期日，期权的Δ越大（如图）

100*100 Puts Delta--91 Days, 10% Volatility, .01% Interest and Using Black-Scholes Futures Style Margin



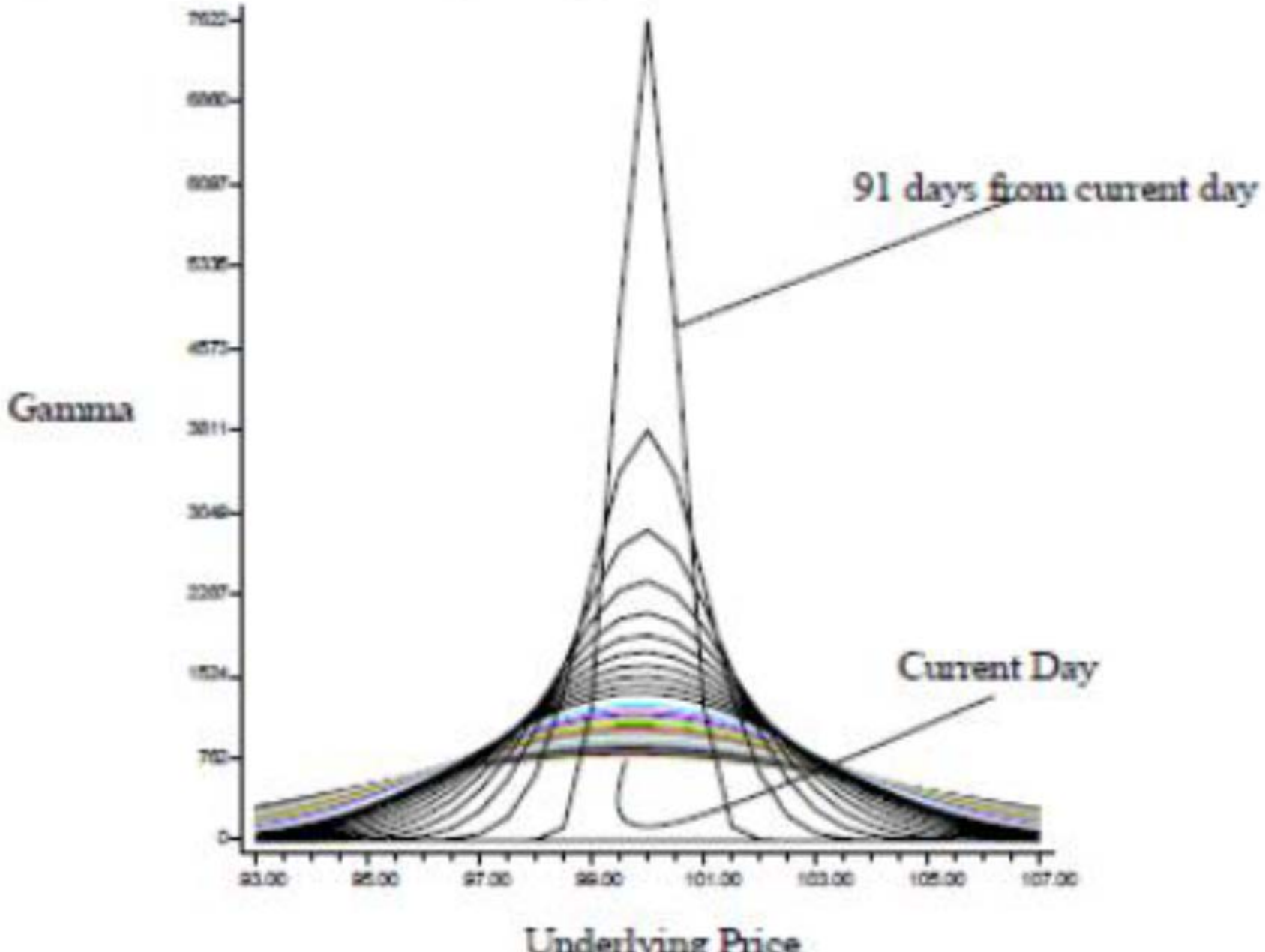
Gamma

- 期权价格对标的资产价格的二阶偏导数
- 度量了期权Delta对标的资产价格变化的敏感性
- 其定义为： $\Gamma = \frac{\partial^2 \Pi}{\partial S^2}$
- 对于欧式期权，Gamma为：

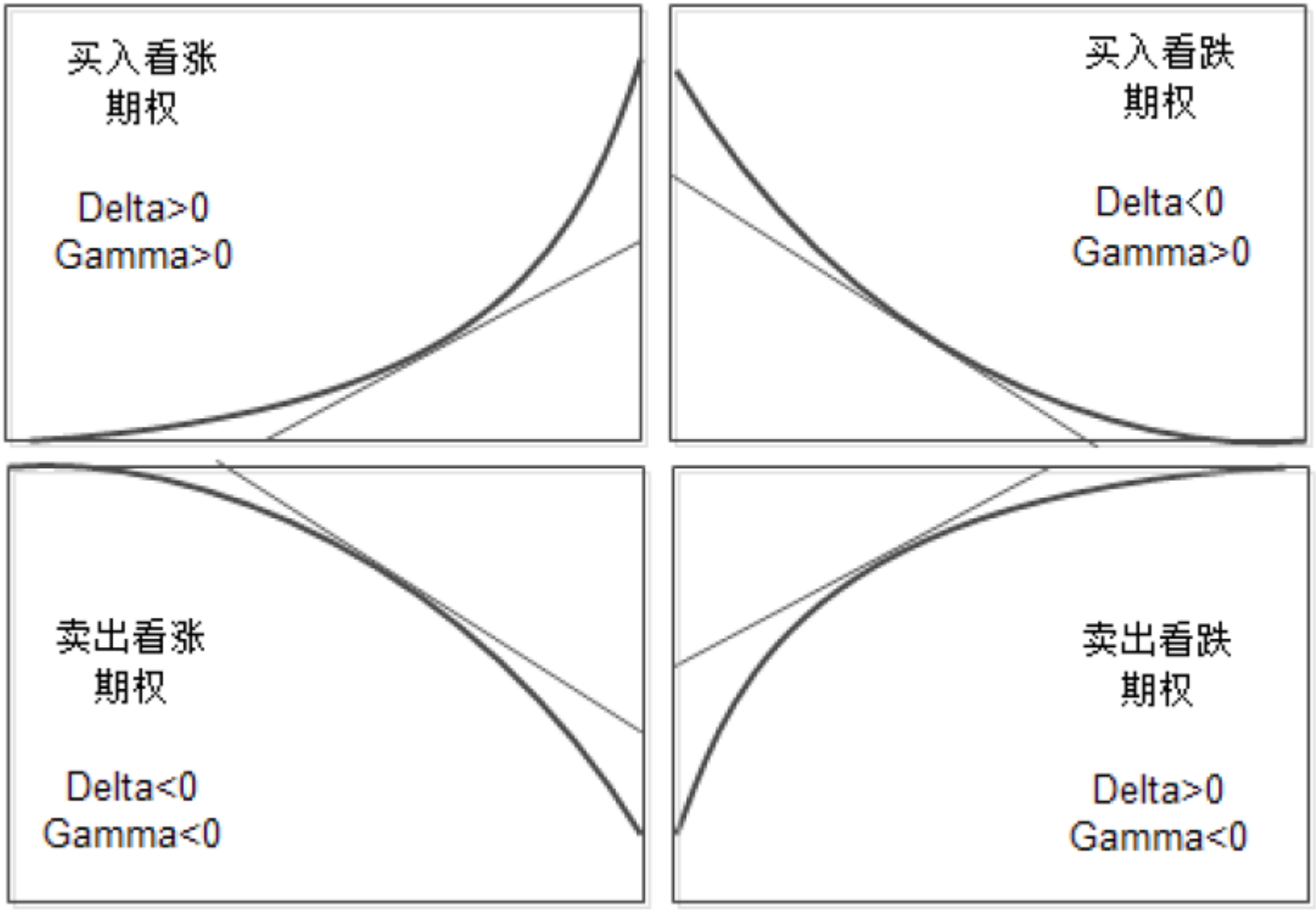
$$\Gamma_c = \Gamma_p = \frac{e^{-qT} N'(d_1)}{S_0 \sigma \sqrt{T}}$$

- 越接近于到期日，gamma越大

100*100 Strike Gamma over Time -- 91 Days, 10% Volatility, .01% Interest and Using Black-Scholes Futures Style Margin



Delta、Gamma 的符号



Theta

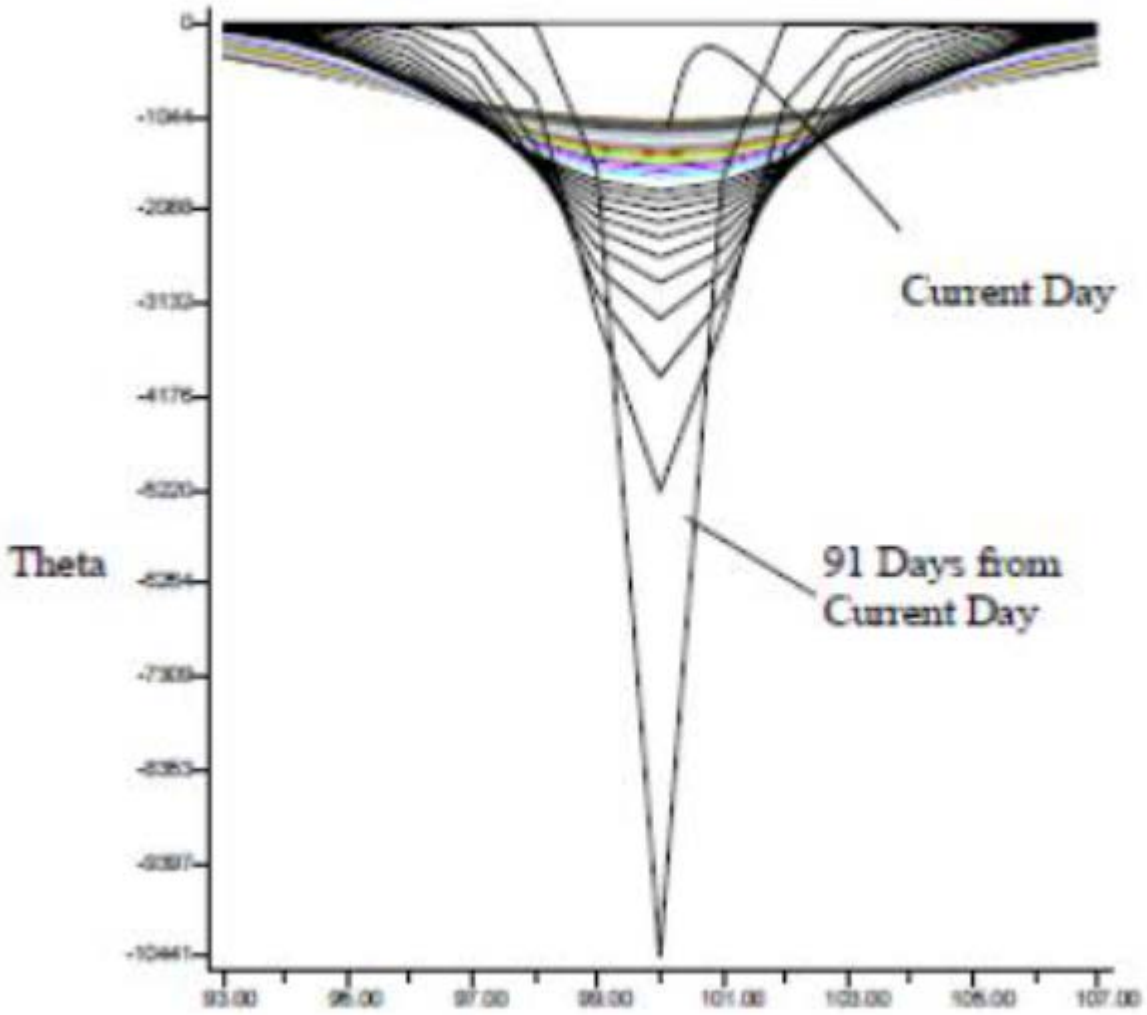
- 期权价格对时间的偏导数
- 度量了期权价格随时间衰减的速度
- 定义为 $\Theta = \frac{\partial \Pi}{\partial \mathrm{t}}$
- 欧式期权的Theta为

$$\Theta_c = -\frac{Se^{-qT}\sigma N'(d_1)}{2\sqrt{T}} + qSe^{-qT}N(d_1) - rKe^{-rT}N(d_2)$$

$$\Theta_p = -\frac{Se^{-qT}\sigma N'(d_1)}{2\sqrt{T}} - qSe^{-qT}N(-d_1) + rKe^{-rT}N(-d_2)$$

- 多头头寸的theta值通常为负
- 随着时间消逝，期权价值在下降
- 下图表示了对于各种价格和到期日的标的资产，看涨期权的theta值的动态特征

100*100 Strike Theta as the Underlying Changes– 91 Days, 10% Volatility, .01% Interest and Using Black-Scholes Futures Style Margin



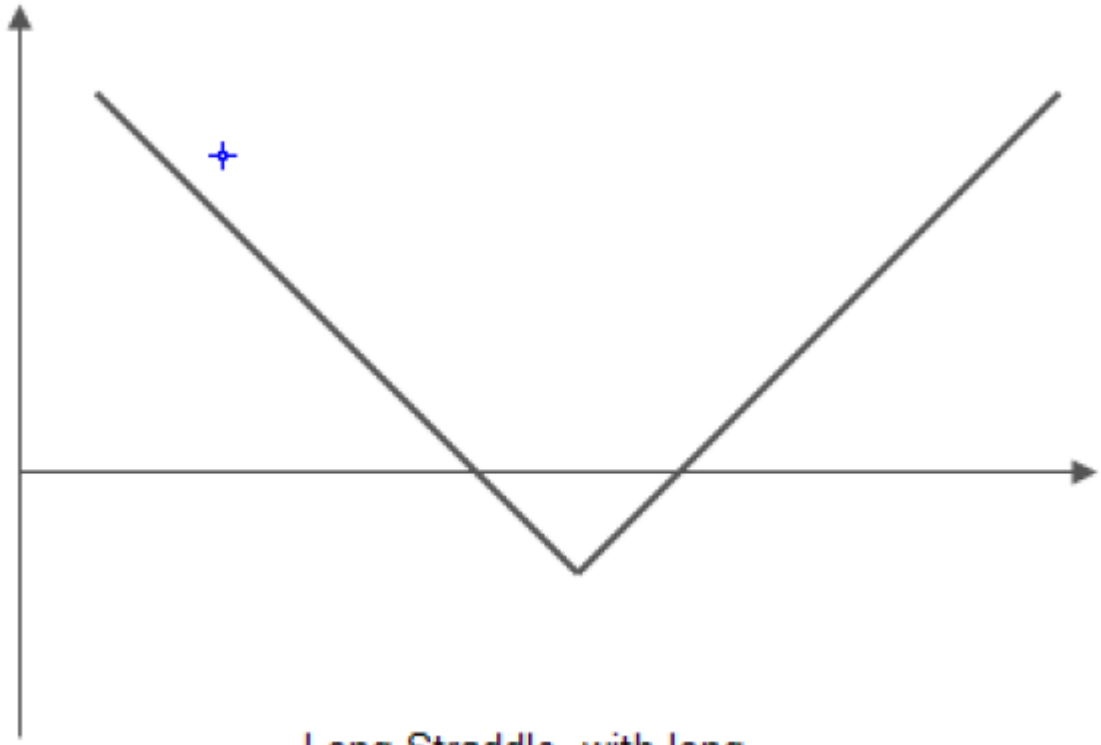
Gamma和Theta的关系

- Delta, Gamma $\Theta + rS\Delta + \frac{1}{2}\sigma^2 S^2 \Gamma = r\Pi$ 有如下关系：

- 在一个delta中性的组合中 delta=0，故

$$\Theta + \frac{1}{2}\sigma^2 S^2 \Gamma = r\Pi$$

- 对于一个delta中性，正gamma投资组合，必定有负的theta
- 此投资组合，赚取价格变动的收益，但随着时间趋于到期日而逐渐减值
- 例：Long straddle



Long Straddle, with long positive gamma and negative theta

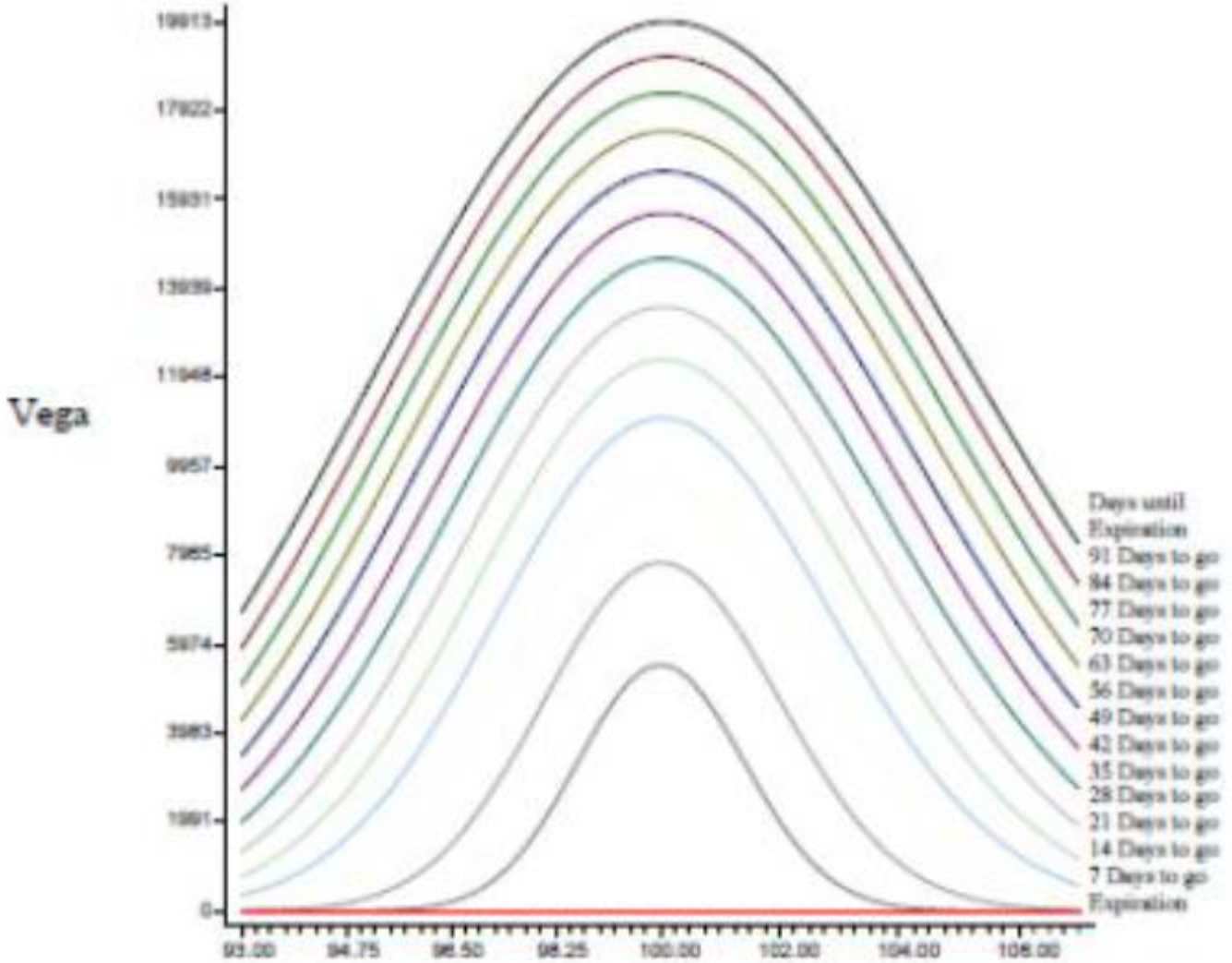
Vega

- 期权价格对资产波动率的偏导数
- 度量了期权价格对标的资产波动率的敏感性
- 定义 $\Lambda = \frac{\partial \Pi}{\partial \sigma}$
- 欧式期权的Vega为

$$\Lambda_c = \Lambda_p = Se^{-qT}\sqrt{T}N'(d_1)$$

- Vega的符号与gamma相同
- 下图给出了long call的vega动态图
  - 平值期权对于波动率最敏感
  - 随着到期日临近，vega逐渐减小
  - 长期的平值期权的vega值最高

100*100 Strike Vega Over Time – From 91 Days to Expiration, 10% Volatility, .01% Interest and Using Black-Scholes Futures Style Margin



Rho

- 期权价格对无风险利率的偏导数
- 度量了期权价格对无风险利率的敏感性
- 定义为 $$\rho = \frac{\partial \Pi}{\partial r}$$
- 欧式期权的Rho为

$$\rho_c = Ke^{-rT}TN(d_2), \quad \rho_p = -Ke^{-rT}TN(-d_2)$$

- 国内的利率自由化程度很低，不展开讨论

期权交易策略简介

- 期权交易策略的选择
- 买入期权策略 Long Option Strategies
- 价差期权策略 Spreading
- 蝶式期权 Butterfly

期权交易策略的选择

Vol\Spot	**Bearish**	**Neutural**	**Bullish**
Bearish	long put	long straddle/strangle	long call
Neutural	bear spread		bull spread
Bullish	short call	short straddle/strangle	short put

买入期权策略 Long Option Strategies

- Purchase of a single option
- or purchase of an option in the form of s a strangle or straddle
- Limited risk and unlimited reward

买入期权的局限性 Limitations of Long

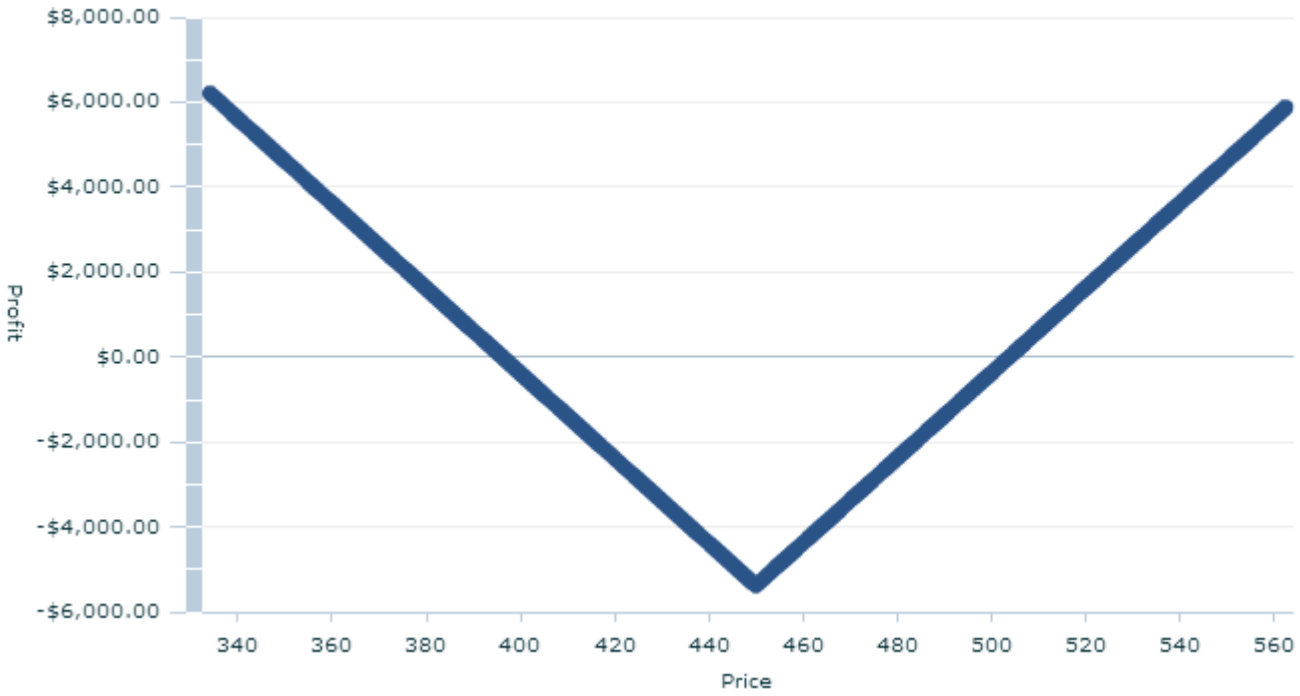
- Time decay
  - Trading @2.20, buys a 2.40 call with 2 months left
  - On the day of expiration, market trading @2.38
  - Loss of the option premium
- 80/20 rule
  - 80% of the all options expires worthless
- Time limit
- Market direction
  - 66% chance of loss immediately after entering the trade

何时买入期权 When to Long Options

- Low volatility
- Extreme prices
- Quiet markets

买入跨式期权 Long Straddle

- When to Use
  - Market will move sharply but do not know the direction
  - Good to use when market has been flat or trading in a narrow range and a breakout is expected
- Profit Profile
  - Unlimited potential in either direction
  - BE on call side = strike price + premium + tran cost
  - BE on put side = strike price - premium - tran cost
- Risk
  - Limited to paid for spread + commission + fees
  - Maximum loss reaches if market is at strike price



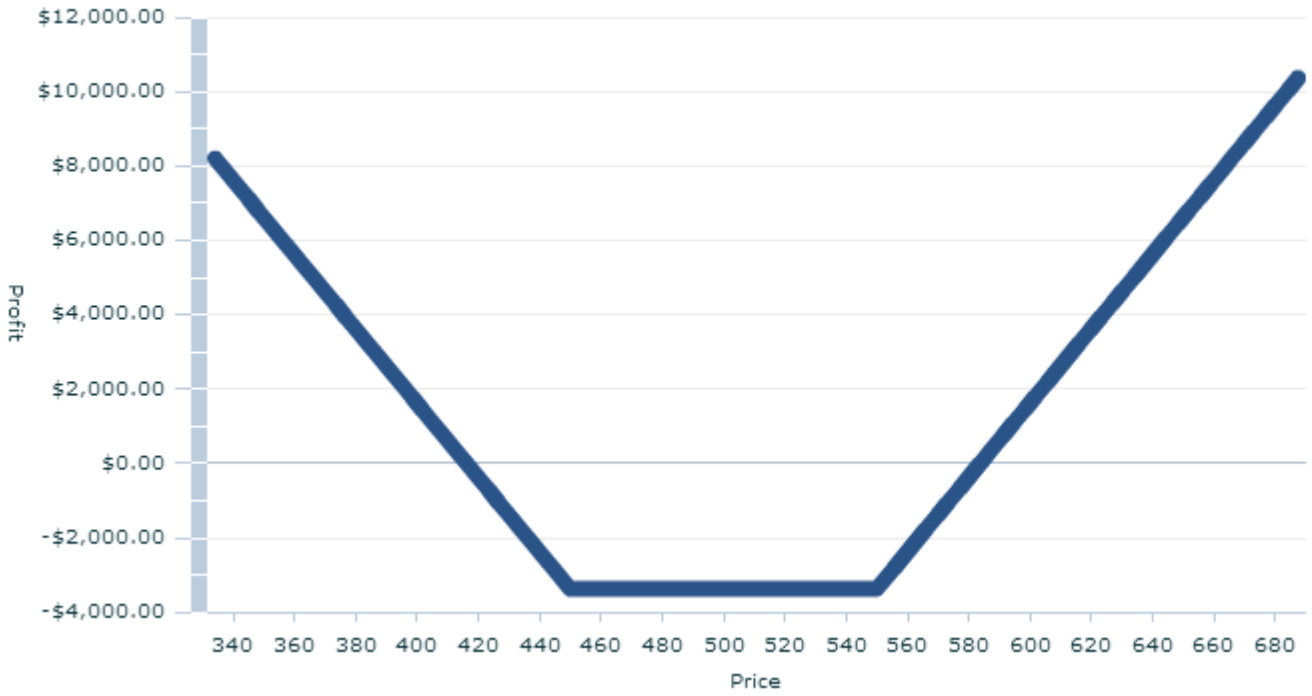
Current Price: $445.52

Price	Profit / Loss	ROI %
334.14	$6,210.00	115.51%
377.97	$1,827.30	33.99%
396.24	$0.00	0.00%
424.10	($2,786.00)	-51.82%
450.00	($5,376.00)	-100.00%
470.23	($3,352.70)	-62.36%
503.76	$0.00	0.00%
516.37	$1,260.70	23.45%
562.50	$5,874.00	109.26%

Second Plot Date

买入鞍式期权 Long Strangle

- When to Use
  - Market is range bound, expect it to break out by making a large move, but direction is hard to predict
  - Similar to straddle, but less expensive, should also be done during times of low volatility and premium
- Profit Profile
  - Unlimited potential in either direction
  - BE on call side = strike price + premium + tran cost
  - BE on put side = strike price - premium - tran cost
- Risk
  - Limited to paid for spread + commission + fees
  - Maximum loss reaches if market is between two strikes



Current Price: $445.52

Price	Profit / Loss	ROI %
334.14	$8,205.00	242.68%
401.96	$1,423.30	42.10%
416.19	$0.00	0.00%
450.00	($3,381.00)	-100.00%
473.34	($3,381.00)	-100.00%
544.73	($3,381.00)	-100.00%
550.00	($3,381.00)	-100.00%
583.81	$0.00	0.00%
616.11	$3,230.40	95.55%
687.50	$10,369.00	306.68%

Second Plot Date

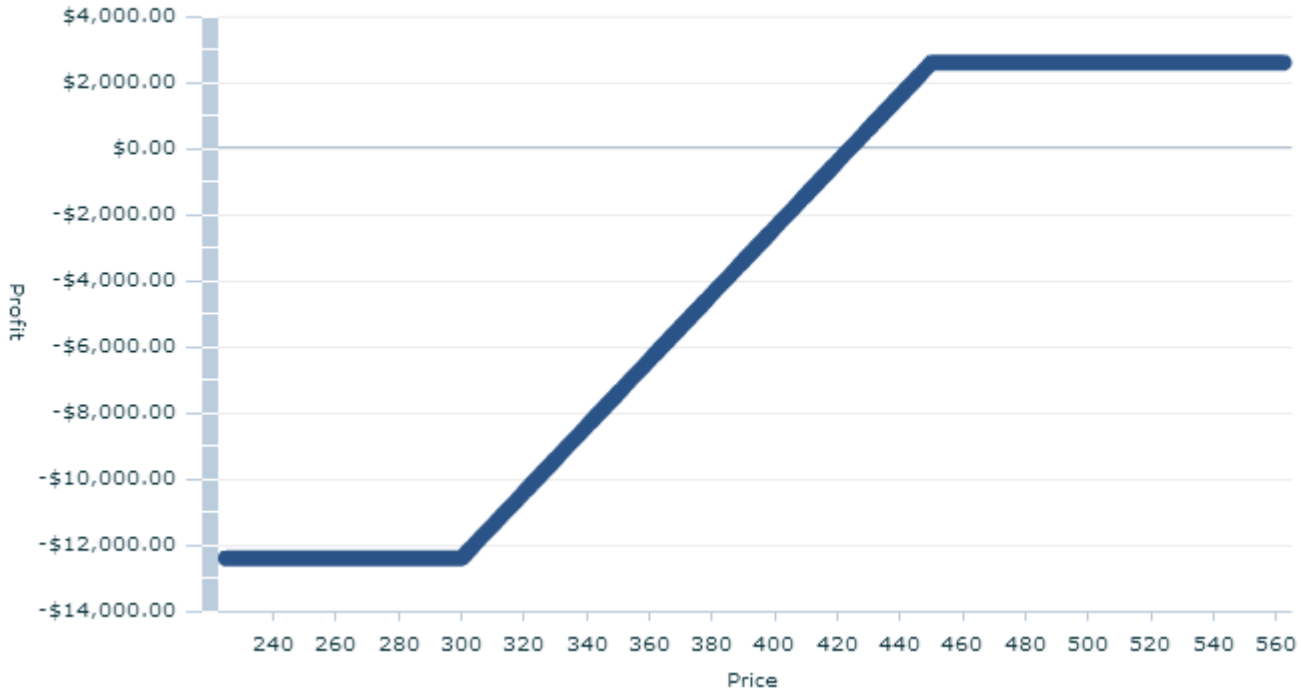
牛市价差期权 Bull Call Spread

- When to Use
  - Believe market is not going down (bullish)
  - Unwilling to accept unlimited loss potential
- Profit Profile
  - Profit limited to premium collected – tran cost
  - Profit maximize if market is at/above the strike price of short put
  - RBE = Short Strike - Net Premium Collected + Cost

- Risk
  - Limited by buying the distant option
  - Maximum loss is difference between short and long option strike prices – premium collected + trans cost
  - Maximum loss occurs when market is at/below long put strike price

Bull Spread

- Long 300 Call/Short 450 Call

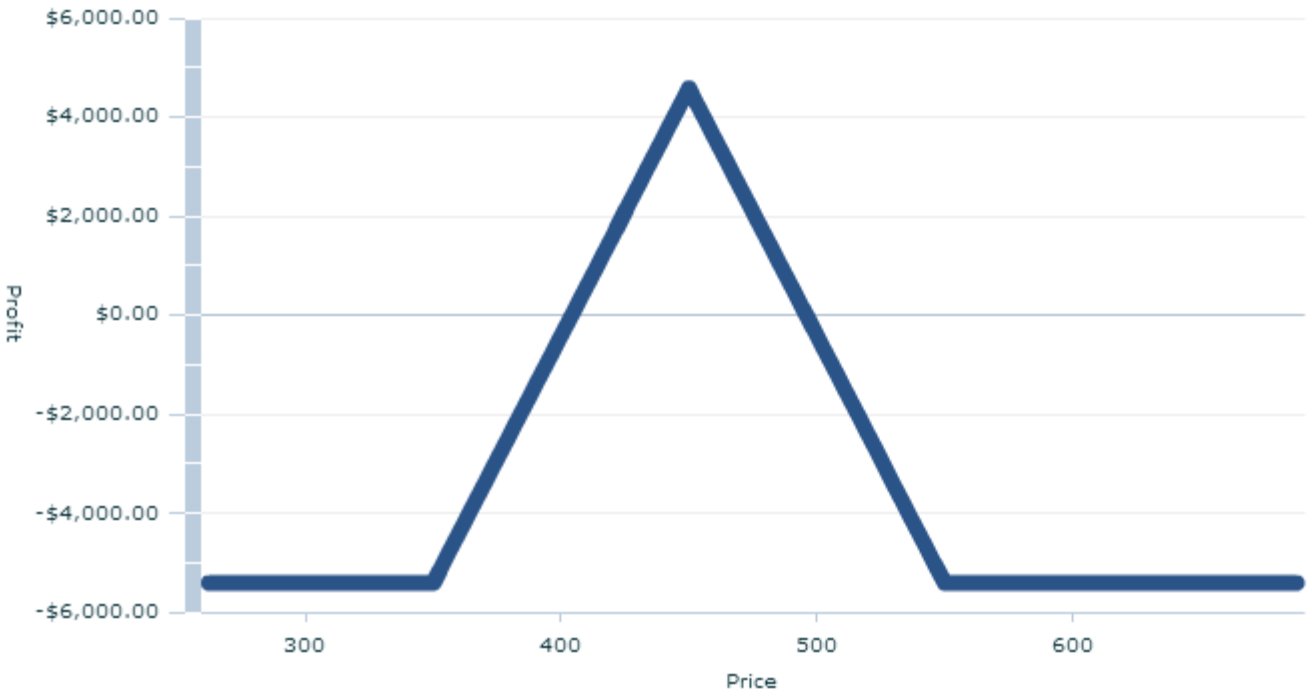


Current Price: $445.52

Price	Profit / Loss	ROI %
225.00	($12,414.00)	-100.00%
289.77	($12,414.00)	-100.00%
300.00	($12,414.00)	-100.00%
357.95	($6,618.50)	-53.31%
424.14	$0.00	0.00%
426.14	$199.60	1.61%
450.00	$2,586.00	20.83%
494.32	$2,586.00	20.83%
562.50	$2,586.00	20.83%

Second Plot Date

Long Butterfly



Current Price: $445.52

Price	Profit / Loss	ROI %
262.50	($5,414.00)	-100.00%
344.07	($5,414.00)	-100.00%
350.00	($5,414.00)	-100.00%
404.14	$0.00	0.00%
429.92	$2,578.40	47.62%
450.00	$4,586.00	84.71%
495.86	$0.00	0.00%
515.78	($1,992.30)	-36.80%
550.00	($5,414.00)	-100.00%
601.64	($5,414.00)	-100.00%
687.50	($5,414.00)	-100.00%

Second Plot Date

应用Greeks管理期权策略
Manage options strategies via Greeks

- Basics
- Spreading
- Portfolio manager's favorites
- Volatility trader

Basics

- Outright call
- Outright put

Spreading

- Bull spread
- Bear spread

Portfolio mangers’ favorites

- Covered call
- Covered put

Volatility Trader

- Straddle
- Strangle